Leveraging Customized Heterogeneous Batteries to Alleviate Low Battery Experience for Mobile Users

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***Abstract*—Even with advances in single-cell batteries, mobile users still experience low battery anxiety. By analyzing 19,855 hours of user behavior, we propose MixMax, a heterogeneous battery system consisting of three complementary battery types tailored to minimizing low battery time. While the heterogeneous battery system offers an opportunity to simultaneously improve capacity and charging speed, one must face non-trivial challenges to design charge/discharge policies during runtime and determine the ratio of enclosed batteries. They are highly dependent on each other, which entails almost infinite candidates for the choice. MixMax simplifies this by reformulating the problem as an optimization problem, breaking it down into manageable sub-problems. However, MixMax still faces the challenge of catering to all users due to their diverse battery usage patterns. To address this, we introduce a customized MixMax that groups users based on their usage patterns and provides tailored battery solutions. In evaluating MixMax, we fabricate coin-cell batteries, develop a precise battery emulator using the fabricated batteries, and prototype MixMax on a real-world smartphone. Our evaluation shows that MixMax reduces low battery time by up to 24.6% without compromising capacity, volume, weight, or user behavior, and its customized version can further reduce it by up to 46.2%.**

***Index Terms*—Battery management systems, heterogeneous battery systems, low battery anxiety, mobile devices.**

Received 7 January 2025; revised 23 July 2025; accepted 14 September 2025. Date of publication 17 September 2025; date of current version 10 December 2025. This work was supported in part by the Ajou University research fund, in part by the National Research Foundation of Korea (NRF) under Grant RS-2024-00438248, and in part by the Ministry of Education under Grant RS-2024-0045198, funded by the Korea government (MSIT). Recommended for acceptance by Jiang Xie. *(Corresponding authors: Jinkyu Lee; Insik Shin.)*

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Digital Object Identifier 10.1109/TSUSC.2025.3611159

I. Introduction

MOBILE devices have become more embedded in our daily lives, playing ever-important roles in our routines. This growing reliance has led individuals to be increasingly sensitive about the remaining energy of batteries in their mobile devices. In particular, a myriad of people report feelings of discomfort or anxiety when their device's battery is low. Worse yet, it becomes more critical when using a smartphone for mobile payment, map, or authentication. Indeed, many existing studies have proved that the so-called *low battery anxiety* [1] is a critical concern for mobile users. Surveys suggest that more than 90% of people suffer from this form of anxiety [2], and low battery anxiety can even lead to behaviors such as asking strangers to charge their devices or significantly reducing device usage to conserve battery energy [1].

A practical and user-friendly way to alleviate low battery anxiety is to reduce the period a device remains in a low battery state, which we call *Low Battery Time (LBT)*. In this respect, this paper aims to minimize the low battery time even without requiring the modification of user behavior on battery charging and discharging. To this end, we first seek to understand users' battery usage patterns by analyzing a total of 19,855 hours of battery usage behaviors collected from 100 mobile users. Our careful analysis suggests two insights for reducing LBT under some typical users' battery usage patterns: *i)* increase charging speed and *ii)* increase battery capacity. However, the physical limitations of single-chemical batteries in mobile devices make it challenging to simultaneously increase capacity and charging speed in the near future [3].

Various studies have been introduced to mitigate low battery anxiety; energy consumption monitoring and analysis tools [4], [5], [6], [7], [8], [9], [10], [11], [12] provide users and software developers with guidance on how to optimize energy consumption. Software-centric optimization approaches such as application-level [2], [5], [7], [13], [14], [15], [16], [17] and system-level [18] introduced various techniques to reconstruct software behavior to reduce energy consumption. As such, a number of studies proposed techniques to minimize energy consumption for a single chemical type of battery. A few studies have proposed new mobile battery systems [19], [20], [21], including multi-cell batteries, beyond a single chemical type of battery, focusing on how to improve specific aspects of the battery performance in terms of capacity and discharging. However, no studies have explored designing new battery systems of multiple chemical types to reduce low battery time.

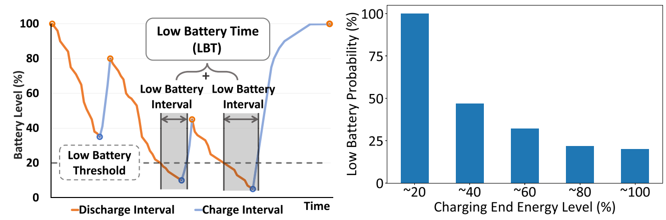
This paper aims at developing a practical method of utilizing multiple types of batteries to mitigate the low battery from this motivation. To this end, we propose MixMax, a novel type of heterogeneous battery system that uses three different types of batteries to jointly embody two approaches, *large capacity* and *high-speed charging*, to reduce LBT. At the core of MixMaxis to find the best solution for its three main components, 1) the composition of different batteries, 2) discharge policy, and 3) charge policy, to collectively minimize LBT without user behavior change. The impact of the three main components on LBT is not straightforward and, further, mutually dependent. It is intractable to find an optimal solution. We propose a practical approach to decompose the problem into sub-problems to reach near-optimal solutions by disentangling the complex dependencies between the main components and solving them step by step. In addition, we propose a customized MixMaxthat provides appropriate battery solutions considering users' battery usage patterns to benefit a broader range of users. We also devise charging/discharging circuits that enable the operation of MixMax.

To evaluate MixMax, we fabricate coin-cell batteries and develop a precise emulator that fully emulates the battery operation by experimenting with the fabricated batteries. We replayed the users' battery usage patterns with the emulator to measure LBT, discovering an overall 24.8% reduction in LBT compared to the users' mobile devices with a single-cell battery. The adoption of customization further accentuated and broadened this benefit, reducing LBT by 46.2%. In addition, we evaluate the effectiveness of MixMax by comparing it with other heterogeneous/multi-cell management solutions [20], [22], [23] and test its applicability by adopting various battery form factors. The evaluation results show MixMax is superior to other design options and applicable to other battery systems with any battery type.

Lastly, we conduct a field test with a demo smartphone that employs heterogeneous batteries according to MixMax. The field test demonstrates the practicality of MixMax by confirming low-cost overhead and operation stability in porting MixMax to the smartphone.

This paper makes the following contributions:

- Derivation of the most important factors to minimize LBT, based on the real battery usage data;
- Design of MixMax, a groundbreaking heterogeneous battery system, with core mechanisms (i.e., the charge/discharge policy and battery ratio optimization), aimed at minimizing LBT;
- Development of a customized MixMax by adapting to individual battery usage patterns, significantly boosting its versatility and performance;
- Fabrication of coin-cell batteries and a real-world demo smartphone, essential for emulator development and field test required by a thorough MixMax evaluation; and
- Extensive evaluation of MixMax and its customized version with real users' battery usage, various battery types consideration, and competitive research comparison.



(a) Example of battery usage behavior and the concept of low battery. (b) Probability of encountering a low battery state based on battery level.

Fig. 1. Analysis and illustration of battery usage patterns and low battery.

The conference version of this article [24] presented a heterogeneous battery system, MixMax, which alleviates mobile users' low battery experience. This updated article addresses a previously unacknowledged limitation of the original MixMax. We have developed a customized MixMax specifically designed to overcome this limitation (in Section V) and evaluated its effectiveness (in Section VII-E). Furthermore, this version has been augmented with additional related work and more extensive discussions on battery-related issues.

II. Motivation

In this section, we examine users' low battery experiences and present key observations to mitigate their impact, offering insights derived from behavioral patterns.

A. Battery Usage Pattern Analysis

We first analyze the daily battery usage patterns of smartphone users in order to understand why and how the users fall into the low battery state. We collect 19,855 hours of battery usage patterns from 100 users, averaging 8.2 days per user, including the chronological order of the battery level (the ratio of the remaining energy to its maximum energy capacity) and charging time. The data of 50 users was collected directly via Android dumpsys [25], and the other 50 users' data was from ExtraSensory Dataset [26], [27], a smartphone sensor-measurement dataset.

A battery usage pattern can be expressed as an alternating sequence of charging and discharging intervals as shown in Fig. 1(a). In each battery usage pattern, we focus on the low battery state where the remaining energy is no larger than the given low battery threshold Δ (to be detailed in Section III-A). A time interval is called *Low Battery Interval* (LBI), if the interval is in the low battery state and starts/ends at which the remaining energy is the same as the low battery threshold as shown in Fig. 1(a). Also, the accumulation of the lengths of all LBIs is called *Low Battery Time* (LBT).

Observation 1. Low battery is pervasive and users want to avoid it: Our battery usage pattern analysis confirms that the low battery is a very general but undesirable problem for mobile device users. During a week, 86 out of 100 users experienced the low battery at least once, and 51 users experienced it five times or more. Each user underwent an average of 1.5 hours of LBT per day, while 18 users stayed in the low battery state for more than 3 hours. Also, users attempted to charge batteries to escape from the low battery state. We observe from the battery usage patterns that users in the low battery state tend to charge their devices more frequently (308%) and longer (37%) than those not.

Observation 2. The more remaining energy, the more chance to avoid the low battery state: Our analysis of user behavior during the discharging intervals, as presented in Fig. 1(b), reveals that the probability of a user experiencing the low battery decreases rapidly as the battery level after charging increases linearly. From this pattern, we derive two effective ways to reduce LBT without changing battery usage patterns: increasing i) the charging speed and ii) the capacity of the battery system.

First, a faster charging speed increments the remaining energy and accelerates the low battery state escape. According to the battery usage pattern analysis, we discovered that 67% of charging intervals end before fully charged, and about one-third of LBT is imposed in charging intervals. This indicates that increasing charging speed can effectively raise the remaining energy of many charging intervals and reduce LBT by getting out of the low battery state earlier.

Second, increasing capacity yields more remaining energy when fully charged. The increased capacity helps reduce LBT if there exists a situation where a user connects the device to a charger even if the battery is already fully charged. Our battery usage pattern analysis disclosed that about 30% of charging intervals belong to the situation; a typical scenario is a user sleeping while charging.

B. Limitation of Other Battery Types

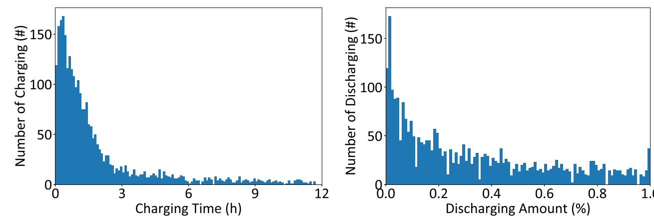
In Section II-A, we observed that most users suffer from low battery, and an effective strategy to address that is increasing capacity and charging speed. We now discuss employing other battery types to increase capacity or charging speed.

Lithium Cobalt Oxide (LCO) battery, the predominant battery type in the mobile industry, has well-balanced capacity and charging speed [28], [29]. However, the physical nature of energy-storing devices like batteries shows an inverse correlation between capacity and charging speed. Thereby, no other current batteries outperform LCO in both aspects; also, it is difficult to develop such a battery in the near future [3].

Then, one may wonder what would happen if a mobile replaces LCO with another battery type to improve one aspect and give up another; however, this approach does not help reduce LBT. We compare an LCO with the following two batteries: Lithium Titanate Oxide (LTO) and Lithium-Sulfur (Li-S). For the comparison, we fabricate the batteries and build a precise battery emulator, which will be detailed in Section VI.

LTO can provide 207% faster charging speed than LCO but has 50% less capacity. Li-S has 99% more capacity than LCO but provides 32% slower charging speed. According to our emulation, LTO and Li-S exhibit 463% and 89% longer LBT, respectively, than LCO.

Observation 3. A battery biased to a single performance aspect is unfit for mobile devices: In short charging/discharging intervals, the LTO can charge more energy by exploiting its fast charging speed; on the other hand, its small capacity is unfavorable for long charging/discharging intervals. Due to the opposite characteristics, the converse holds for Li-S. While each of the two batteries fits either short or long intervals, mobile users show a complex usage pattern where long and short intervals are mixed. Using the battery usage patterns from Section II-A, Fig. 2(a) represents the distribution of charging time (i.e., the time duration of each charging interval), and Fig. 2(b) represents the distribution of discharging amount (i.e., the variance in battery level during each discharging interval). Both distributions indicate that the users not only frequently charge/discharge the battery for short intervals but also considerably charge/discharge for long intervals corresponding to the long tails of the distributions. Due to this incongruity of the battery usage pattern, it cannot yield LBT reduction to employ either LTO or Li-S instead of LCO battery.



(a) Charging time distribution. (b) Discharging amount distribution.

Fig. 2. Charging and discharging behaviors of 100 mobile users.

As such, at the current level of battery material engineering, no other battery type can further reduce LBT by increasing both capacity and charging speed simultaneously. Therefore, this paper proposes a novel mobile battery system, MixMax, that utilizes a multi-cell heterogeneous battery system to alleviate low battery time instead of a single-cell battery system.

III. System Overview

A. Problem Statement

In this paper, we aim to design a heterogeneous battery system, MixMax, which is designed to alleviate the low battery time of smartphones based on typical battery usage patterns. This raises several issues that need to be explored as follows:

Battery Types: As discussed in Section II, we seek to extend the performance of the prevailing battery type in the battery industry by increasing capacity and improving charging speed simultaneously. To this end, we construct MixMax with the following three battery types (see Fig. 3) sorted by a descending order of power density (that determines charging speed per volume) or an ascending order of energy density (that determines capacity per volume):

- *A-type* that exhibits higher power density but lower energy density than B-type (*e.g.*, LTO),
- *B-type* that is widely used in the state-of-the-art mobile devices (*e.g.*, LCO), and
- *C-type* that exhibits higher energy density but lower power density than B-type (*e.g.*, Li-S).

Note that although we mainly focus on the trade-off between power density and energy density, the cycle life of the B-type is shorter and longer than A- and C-type, respectively.

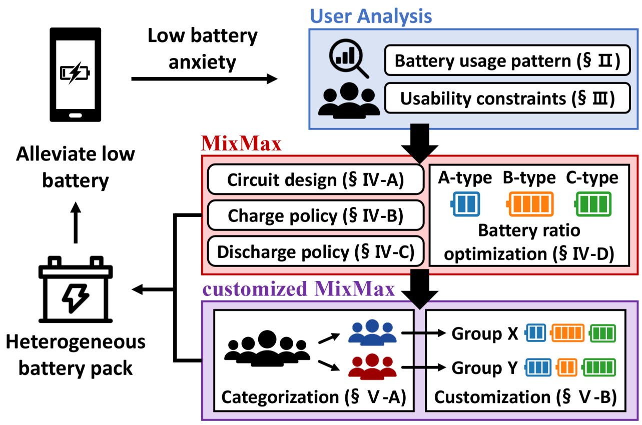


Fig. 3. Overview of MixMax and customized MixMax.

Battery Ratio: Once the battery types are chosen, the next issue is determining the ratio of each type of battery in terms of physical volume. Let denote the ratio of A-, B-, and C-type batteries as R_A:R_B:R_C. According to the R_A:R_B:R_C, the absolute physical volumes of the three battery types are calculated from any given total battery volume budget.

Charge & Discharge Policies: A heterogeneous battery system introduces new issues, which are not considered in a single-cell battery system. It needs to determine which types of battery to use for charging and discharging. For example, when all three types of batteries are available for charging or discharging, one may strategically choose some or all of them for performance optimization.

We formally state the *optimization problem* to be solved by MixMax as follows.

Given some typical battery usage patterns of mobile devices,

Determine (i) the battery volume ratio, (ii) charge policy and (iii) discharge policy of a three-type heterogeneous battery system,

In order to minimize the *low battery time* (LBT), where LBT is the duration in which the sum of the remaining energy in A-, B- and C-type batteries is less than some given threshold Δ,

Subject to the given volume budget, the minimum capacity and the maximum aging.

In order to design a heterogeneous battery system that outperforms the state-of-the-art mobile battery (i.e., LCO in B-type), the usability constraints in the optimization problem are set to the volume, capacity and aging of a representative B-type battery.

In the constraints, the given volume budget[1] is responsible for the physical deployment of the heterogeneous batteries of MixMax in mobile devices, while the minimum capacity enables it to satisfy users' demand for the maximum energy. Also, the maximum aging ensures no more capacity degradation from battery aging. It is worth noting that although we do not explicitly consider the constraints of weight and cost, our solution to the above optimization problem is comparable to existing B-type batteries in terms of weight and cost to be discussed in Section X.

Similarly, the threshold Δ can also be set to the energy level at which most smartphones start displaying low battery warnings. This is equivalent to 15% of the capacity of the B-type battery for Android smartphones and 20% for iOS smartphones. In this paper, we set the threshold to 20% by referring to other studies [1], [2].

After solving this optimization problem and designing MixMax, we further improve its performance by developing a customized version (illustrated in Fig. 3) that is more tailored to individual users' battery usage patterns. This will be detailed in Section V.

B. Challenges and Approach Overview

Challenges: We address the problem of reducing LBT by determining how to charge/discharge and compose the three different battery types. This problem necessitates the design of a heterogeneous battery system and the porting of the designed system into an actual system.

Designing a heterogeneous battery system entails interdependent sub-problems. One can easily expect that increasing the ratio of A-type battery results in faster charging speed. However, it is quite difficult to figure out the exact charging speed of the heterogeneous batteries. Unlike a single-cell battery system that typically has a constant maximum charging speed, a heterogeneous battery system exhibits the unique characteristics that its maximum charging speed varies as its design components: charge/discharge policies and battery ratio. Additionally, the impact of one component on the charging speed depends on the design of other components. Therefore, it is challenging to understand the complicated effects of these interdependent components and to design them in favor of reducing LBT. Even worse, reducing LBT requires deeply considering users' battery usage patterns.

Even if we develop the design solution, it remains to be seen whether the heterogeneous battery system can be deployed to mobile systems from a practical point of view. A mobile system must be capable of supporting electrical functions like switching and converting in order to apply the heterogeneous battery system.

Design Overview: In this paper, we divide and conquer the above challenges. The problem of reducing LBT by the heterogeneous battery system is divided into design and practical aspects, and the design problem is solved step by step in the order of less affected by the users' battery usage pattern. From a practical aspect, we first establish MixMax-support circuits since existing smartphone circuits cannot operate heterogeneous battery systems. Then we design the charge/discharge policies, which are highly correlated with the battery properties, and based on the policies, we optimize the battery ratios considering the user battery usage pattern. For the last step, we verify its practicality with real system implementation.

[1]Note that the given volume budget considers the total volume of batteries only. Operating a heterogeneous battery system necessitates additional components which incur extra volume, but their volumes are tiny and even non-deterministic at this stage [30], [31], [32]. Therefore, the stated optimization problem considers the volume of batteries. Details will be discussed in Section VIII.

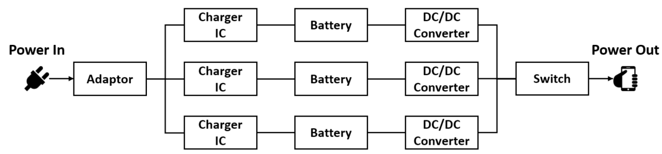


Fig. 4. Schematic diagram of MixMax circuitry.

IV. MixMax System Design

In this section, we design MixMax that minimizes LBT, which consists of three components: charge policy, discharge policy, and battery ratio optimization. We begin by devising charging/discharging circuits to support the operation of MixMax. Based on this support, we first develop an ideal charge policy that is independent of other components and then design a discharge policy that maximizes the charged energy under the charge policy. Finally, we determine the battery ratio that minimizes LBT under the charge/discharge policies.

A. Charging & Discharging Circuits

Operating multiple types of batteries requires special circuits. MixMax involves heterogeneous batteries, and it is not trivial to manage them due to their different electrical characteristics. Traditional smartphone circuits targeting single-cell battery system cannot handle them. To operate heterogeneous battery systems, there are some circuit design options available [20], [33]. However, a circuit configuration that is overly complex or too simplistic will cause operation failure, higher costs, and power loss, or impose constraints on designing the core components of MixMax (i.e., charge/discharge policies and battery ratio). Hence, it is crucial to select the appropriate circuitry for MixMax based on its functionality requirements. We refer to existing circuit design options and choose the most suitable circuitry for MixMax. Our proposed charging and discharging circuits for MixMax are conceptually depicted in Fig. 4. With the support of the circuitry, we further design other components of MixMax.

MixMax's charging circuits charge heterogeneous batteries separately in parallel. When power is inputted and undergoes AC/DC conversion at a charger adaptor, MixMax needs to charge its individual batteries with the given power input. With the given single power input, it is necessary to individually charge each battery as each battery has its own charging characteristics (e.g., current/voltage limits). To this end, MixMax places individual charger integrated circuit (IC) with each battery (illustrated in Fig. 4), which converts and manages current/voltage and the charging process. Since this approach is not much different from traditional smartphone charging circuits, it does not incur critical issues or technical challenges.

The discharging circuitry of MixMax distributes a given discharge load to heterogeneous batteries. When a power load is given by the user behavior (the rightmost in Fig. 4), MixMax discharging circuits distribute that load to batteries. In detail, MixMax discharges batteries one by one and switches the discharging among batteries at a high frequency in a round-robin manner. One can adjust the granularity and respective discharging load of batteries with the switching frequency. Additionally, as the power out to the smartphone must have a specific voltage range, the different output voltages of different batteries are converted by DC/DC converters. The described switching approach was originally proposed by Badam et al. [20], and is known to have high power efficiency while demanding few numbers of circuit components. Unlike Badam's work, since MixMax does not require a charge migration functionality that transfers power between batteries sacrificing significant power loss, MixMax 's circuitry is much simpler.

The devised circuits, especially discharging circuits, can affect other components of MixMax, from its power efficiency to design choices. However, we confirm that such effects are not considerable from our field test, which will be discussed in Section VIII. Thus, we further design other components of MixMax on top of the proposed circuitry.

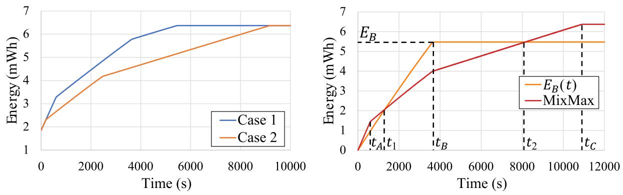
B. Charge Policy

As MixMax deploys heterogeneous batteries, MixMax involves new design issues that do not exist in the single-cell battery system. One of the key issues is how to charge each of the three batteries to minimize LBT. Once we decide the charge policy, we can explore the charging behavior of MixMax, which can be used to determine other MixMax components.

Different from the single-cell battery system, MixMax needs to determine the charging speed of the individual batteries within their different maximum charging speed. While MixMax, for example, may apply the maximum charging speed to A-type, no charging to B-type, and half of the maximum charging speed to C-type, it does not help LBT reduction to deliberately slow down the charging speed of any batteries. This is because, such a slower charging speed yields the lower remaining energy in MixMax, which always has a negative impact on LBT.

Therefore, we let MixMax use the *best-effort charge policy*, which charges all *chargeable* batteries (i.e., batteries that are not fully charged yet) with their own maximum charging speed. When charging, MixMax assumes that the charger can support sufficient power to allow all batteries to charge at their own maximum speed (which is a typical situation where the charger is connected to the power outlet). If not (*e.g.*, when using the charger connected to a laptop), MixMax distributes the given budget of power to the three batteries proportional to their maximum charging speed. As a result, MixMax always charges all the chargeable batteries proportional to their own maximum charging speed, thereby ensuring that the intermediate charging state remains consistent across time and conditions, which helps in designing other components.

When the best-effort charge policy is applied, each battery has a different capacity and charging speed, so the full charging time of batteries varies. This characteristic entails a *multi-stage charging speed* in MixMax. The charging speed of MixMax at a time instant is the sum of the charging speed of the batteries being charged, so the charging speed of MixMax decreases whenever one battery is fully charged, to be detailed in Section IV-D with Fig. 5(b).



(a) Charging MixMax with two different energy distributions under the same total remaining energy.

(b) Multi-stage charging speed of MixMax, compared with the charging speed of a single B-type battery.

Fig. 5. Key charging characteristics of MixMax.

We also find the multi-stage charging speed varies according to the distribution of remaining energy in each battery at the start of the charging interval, even if the total energy is fixed. For example, Fig. 5(a) shows two different initial energy cases when the energy of the MixMax is charged from 1.8 mWh to 6.4 mWh; one is the case where the initial energies of A-, B-, and C- types are 0 mWh, 0 mWh, and 1.8 mWh, and another is the case where those are equally distributed (i.e., 0.6 mWh each). As shown in the figure, the charging speed of MixMax highly depends on the distribution of energy in each battery type. This finding indicates the importance of the discharge policy that determines the distribution, to be discussed in the following subsection.

C. Discharge Policy

We develop a discharge policy that maximizes charging speed. As confirmed in Sections II-A and IV-B, increasing the charging speed can reduce LBT, and the charging speed varies depending on the discharge policy. Thus, a discharge policy to be developed should be designed in order to increase the charging speed in the subsequent charging interval.

Considering the charging speed of MixMax is maximized when all batteries are being charged simultaneously, it is desirable to develop a discharge policy to charge multiple batteries simultaneously. MixMax's charging speed slows down whenever one battery is fully charged, losing that battery's charging speed. Therefore, we make MixMax discharge the battery that will be fully charged at the earliest time instant under the best-effort charge policy proposed in Section IV-B. In other words, our discharge policy delays the earliest time for one of the battery types to be fully charged as late as possible (i.e., maximizing the minimum full charging time). We call this discharge policy *MaxiMin*, and it always ensures the optimal[2] fastest charging speed in a subsequent charging interval. Note that the MaxiMin discharge policy works with any multi-cell/heterogeneous battery system, although MixMax employs three battery types.

Let $T(X)$ denote the time to fully charge the X-type battery[3] (where X can be A, B or C) when MixMax is charged according to the best-effort charge policy. We first check whether a battery must be discharged or not. The X-type battery is flagged to be discharged (denoted by $flag_X$), only when it has the minimum $T(X)$ among A-, B- and C-type batteries and is dischargeable, as follows.

$$flag_X = \begin{cases} 1, & \text{if } T(X) = min(T)\ \&\ X \text{ is dischargeable,} \\ 0, & \text{otherwise.} \end{cases}$$

In the case of existence of multiple non-zero flags, the discharge power load is distributed according to their corresponding charging speed S_X. Let $DL^+(t)$ denote the total discharge power load of MixMax with the A-, B- and C-type batteries at t. Then, the discharge power of the X-type battery at t (denoted by $DL_X(t)$) can be calculated as follows.

$$DL_X(t) = \frac{S_X \cdot flag_X}{\sum_{i=A,B,C} S_i \cdot flag_i} \cdot DL^+(t).$$

The following lemma explains an optimal property of the MaxiMin discharge policy.

Lemma 1: The MaxiMin discharge policy along with the best-effort charge policy always charges more (or the same) energy than any other discharge policy along with the best-effort charging during $[0, t)$, where 0 and t are the beginning and arbitrary end time of a subsequent charging interval.

Proof: Suppose that MixMax discharges three battery types in a given discharging interval by discharge policy DP and charge them in $[0, t)$. Let T_X^{DP} and $C_{\{X,Y,Z\}}^{DP}(0, t)$ denote the required time to fully charge X-type battery of MixMax and the total charged energy in $\{X, Y, Z\}$-type batteries of MixMax in $[0, t)$, respectively. Lemma 1 implies the following statement holds for any t and discharge policy ANY (For an abbreviation, we denote MaxiMin as MM):

Statement: When batteries are discharged by MaxiMin, let m_n denote a battery with the n-th shortest full charging time. That is, $T_{m_1}^{MM} \leq T_{m_2}^{MM} \leq T_{m_3}^{MM}$ holds for $m_1 \neq m_2 \neq m_3 \in \{A, B, C\}$. Then, the following holds:

$$C_{\{A,B,C\}}^{MM}(0, t) \geq C_{\{A,B,C\}}^{ANY}(0, t). \text{(ST)}$$

We now prove ST holds for the following individual cases:

- *Case 1:* $t <= T_{m_1}^{MM}$. As all the three batteries are charged until t with best-effort, ST holds.
- *Case 2:* $T_{m_1}^{MM} < t <= T_{m_2}^{MM}$. Since $T_{m_1}^{MM} < T_{m_2}^{MM}$ holds, in this case, either m_1 is fully discharged or the total discharge energy load has been already met. Therefore, MaxiMin could not have discharged the battery (m_1) further during the discharging interval. Therefore, the following inequality holds:

  $$C_{\{m_1\}}^{MM}(0, t) \geq C_{\{m_1\}}^{ANY}(0, t).$$

  Since $\{m_2, m_3\}$-type batteries are charged until t with best-effort, the following inequality holds:

  $$C_{\{m_2,m_3\}}^{MM}(0, t) \geq C_{\{m_2,m_3\}}^{ANY}(0, t).$$

  Therefore, ST holds.
- *Case 3:* $T_{m_2}^{MM} < t <= T_{m_3}^{MM}$. As in Case 2, the following two inequalities hold:

  $$C_{\{m_1,m_2\}}^{MM}(0, t) \geq C_{\{m_1,m_2\}}^{ANY}(0, t), \text{ and}$$

[2]The optimality holds under the assumption that the amount of discharged energy is unchanged regardless of the discharge policy. Effects such as battery resistance and rate capacity effects are ignored here, but considered later in evaluation.

[3]$T(X)$ is estimated considering charger behavior (*e.g.*, CCCV charging).

$$C^{MM}_{\{m_3\}}(0,t) \geq C^{ANY}_{\{m_3\}}(0,t).$$

Therefore, ST holds.

- *Case 4:* $T^{MM}_{m_3} < t$. As all batteries are fully charged at $T^{MM}_{m_3}$, ST holds.

Therefore, $C^{MM}_{\{A,B,C\}}(0,t) \geq C^{ANY}_{\{A,B,C\}}(0,t)$ holds for any t. ■

While the proposed MaxiMin results in the optimal fastest charging speed under the assumption of ignoring some internal battery characteristics, Section VII will evaluate MaxiMin under realistic environments without the assumption.

D. Battery Ratio Optimization

In this section, we determine the battery ratio R_A:R_B:R_C that minimizes LBT under the optimal charge/discharge policies developed in Sections IV-B and IV-C, where R_A, R_B and R_C (each ≤ 1.0) denote the volume proportion for the A-, B- and C-type batteries in MixMax, satisfying $R_A + R_B + R_C = 1.0$. It is challenging to determine the battery ratio because 1) the relationship between the battery ratio and LBT depends on several factors (such as charge/discharge pattern and the performance trade-off among different battery types) in a complicated manner and 2) the problem has numerically infinite search space. To address the challenges, we perform the following steps (S1–S3).

- *Overview of S1:* Based on analyzing important physical characteristics of MixMax that are independent of users' battery usage pattern, we limit the range of the battery ratio, and derive an intuition of how to decompose the problem of determining the battery ratio.
- *Overview of S2:* By establishing a model that predicts the expected LBT using users' battery usage pattern, we suggest decomposing the problem into (P1) finding the relative ratio between R_A and R_C under given R_B (P2) determining R_A, R_B and R_C under given R_A/R_C.[4]
- *Overview of S3:* Utilizing properties derived from S1 and S2, we determine the battery ratio in a systematical manner, for the actual battery usage pattern with the proposed charge/discharge policies.

S1: The battery ratio determines important physical characteristics of MixMax: charging speed, capacity and power output. As a first step, we analyze the charging speed of MixMax. Fig. 5(b) shows the amount of accumulated energy stored in MixMax in $[0,t)$, which is denoted by $E_{\mathsf{MixMax}}(t)$ in (1). In the equation, E_X denotes the maximum energy to be stored in the single X-type battery that has the same volume as MixMax, while S_X denotes the maximum charging speed of the single X-type battery (where X can be A, B or C). Therefore, the maximum energy to be stored in the X-type battery of MixMax and the maximum charging speed of the X-type battery of MixMax can be calculated as $R_X \cdot E_X$ and $R_X \cdot S_X$, respectively. As shown in Fig. 5(b) and (1), MixMax exhibits the *multi-stage charging speed* behavior with t_A, t_B and t_C at which the A-, B- and C-type batteries are fully charged.

$$E_{\mathsf{MixMax}}(t) = \begin{cases} (R_A \cdot S_A + R_B \cdot S_B + R_C \cdot S_C) \cdot t, & 0 \leq t < t_A, \\ R_A \cdot E_A + (R_B \cdot S_B + R_C \cdot S_C) \cdot t, & t_A \leq t < t_B, \\ R_A \cdot E_A + R_B \cdot E_B + R_C \cdot S_C \cdot t, & t_B \leq t < t_C, \\ R_A \cdot E_A + R_B \cdot E_B + R_C \cdot E_C, & t_C \leq t. \end{cases} \quad (1)$$

In Fig. 5(b), we also plot $E_B(t)$, the amount of cumulative stored energy in the single B-type battery that has the same volume as MixMax in $[0,t)$. If the battery is not fully charged, $E_B(t) = S_B \cdot t$; otherwise, $E_B(t) = E_B$. Then, depending on R_A, R_B, and R_C, time instants t_1 and t_2 exist where the cumulative energy in MixMax is the same as that in the single B-type battery (i.e., $E_{\mathsf{MixMax}}(t) = E_B(t)$), as shown in the figure. We observe that the cumulative energy in MixMax is larger than that in the single B-type battery in $[0,t_1)$ and $[t_2,t_C)$, and the converse holds in $[t_1,t_2)$.

Although it seems very complex how the battery ratios R_A, R_B and R_C affect $E_{\mathsf{MixMax}}(t)$, we discover two important properties. First, if we focus on the time instants t_1 and t_2, which determine whether MixMax exhibits worse or better performance than the corresponding single B-type battery in terms of the cumulative stored energy, we can arrange them by solving the equation $E_{\mathsf{MixMax}}(t) = E_B(t)$. Then the following formulas imply that t_1 and t_2 depend on the relative ratio between R_A and R_C, but not on R_B.

$$t_1 = R_A \cdot E_A / (R_A \cdot S_B + R_C \cdot S_B - R_C \cdot S_C),$$
$$t_2 = (R_A \cdot E_B + R_C \cdot E_B - R_A \cdot E_A)/(R_C \cdot S_C).$$

Second, if we focus $E_{\mathsf{MixMax}}(t) - E_B(t)$, its amount depends on R_B for a given relative ratio between R_A and R_C. The two properties are related to P1 and P2 as follows: solving P1 corresponds to determining the interval length of $[0,t_1)$ and $[t_2,t_C)$ in which $E_{\mathsf{MixMax}}(t) > E_B(t)$ holds and $[t_1,t_2)$ in which $E_{\mathsf{MixMax}}(t) < E_B(t)$ holds, while solving P2 under the solution of P1 corresponds to determining the amount of the difference between $E_{\mathsf{MixMax}}(t)$ and $E_B(t)$. Hence, we try to decompose the problem of determining the battery ratio into P1 and P2, to be justified more rigorously in S2.

When it comes to the maximum capacity of MixMax, it was already derived in the last line of (1). Applying the constraint of the problem statement, which is the capacity of MixMax no less than E_B, we derive $R_A/R_C \leq (E_C - E_B)/(E_B - E_A)$, yielding a range of the ratio as $R_A/R_C \leq 2.0$.

Finally, each battery type should be capable of supplying the maximum power load even when only a single battery type in MixMax has remaining energy. By applying the maximum power load to the maximum discharge power of each battery type, we derive a lower bound of each battery ratio, yielding $R_A \geq 0.05$, $R_B \geq 0.09$, and $R_C \geq 0.18$.

S2: We now investigate how the battery ratio changes LBT. To this end, we establish a model that predicts the LBT trend according to the battery ratio based on the users' battery usage pattern data. To address the complexity issue for the model, we choose a representative situation where the charging starts at a 0% energy level of MixMax, by considering the followings: 1) the charging starts at 0–20% energy level most frequently, 2)

[4]Note that P2 is equivalent to determining R_B under the solution of P1 since $R_A + R_B + R_C = 1.0$ holds.

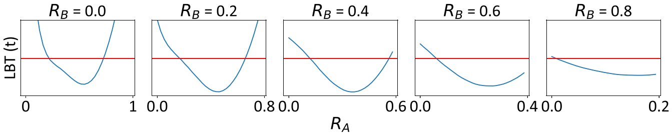


Fig. 6. Expected LBT of MixMax with varying R_A and R_B (blue line), compared to a single B-type battery (red line).

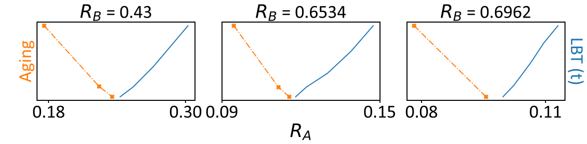


Fig. 7. Objective function values during ratio optimization. It returns aging (orange) when the evaluated aging is greater than the B-Type and returns LBT (blue) otherwise.

the probability of entering the low battery state is lower if the charging starts at a higher battery level, and 3) the situation makes it possible to calculate the remaining energy by (1) without considering the initial energy distribution of each battery type in MixMax. Then, the expected LBT under the situation can be calculated by multiplying the probability of entering the low battery state and the distribution of the low battery interval's length, both of which can be derived from the users' battery usage patterns according to the remaining energy.

Fig. 6 shows the expected LBT of MixMax according to the model, under varying R_A (as x-axis) and R_B (shown in different sub-figures). Note that since $R_A + R_B + R_C = 1.0$ holds, R_C is automatically determined if R_A and R_B are fixed; therefore, each sub-figure also represents the expected LBT according to different R_A/R_C for given R_B. We observe two important properties of the expected LBT from the model. First, once we fix R_A/R_C, the expected LBT is convex with respect to R_B. For example, for given $R_A/R_C = 1.0$, the expected LBT is minimized with $R_B = 0.2$ (where $R_A = R_C = 0.4$ in the second sub-figure) or $R_B = 0.4$ (where $R_A = R_C = 0.3$ in the third graph, and it increases as R_B converges to 0.0 or 1.0. Second, once we fix R_B (i.e., focusing on a single sub-figure), the expected LBT is also convex with respect to R_A/R_C. For example, with $R_B = 0.2$ in the second sub-figure, the expected LBT is minimized with $R_A = R_C = 0.4$; as R_A/R_C deviates from 1.0, the expected LBT increases.

The properties suggest the following guidelines for solving the problem of determining the battery ratio, to be utilized in S3. First, it is feasible not only to solve P1 under a given solution of P2, but also to solve P2 under a given solution of P1. Second, when solving P1 and P2, we can efficiently find the solution using the convexity.

S3: We now solve the optimization problem of finding the battery ratio that minimizes LBT for the actual battery usage pattern with the charge/discharge policies proposed in Sections IV-B and IV-C, which entails the following challenges. First, actual LBT (not derived by the model, but obtained by experiment/emulation) is not a closed-form function of the battery ratio, disallowing mathematical derivation of the battery ratio that minimizes LBT. Second, it takes a long time to obtain actual LBT of the actual battery usage pattern by experiment/emulation, even for a single instance of the battery ratio. Third, the constraint of aging (not larger than the single B-type battery) in the problem statement should be considered along with LBT minimization.

To this end, we develop an empirical optimization process as follows. First, we split the actual battery usage pattern data of 100 users into training and test data at 7:3, and we use only the training data for the optimization.

Second, to address the aging constraint, we design the objective function such that it evaluates both LBT and aging for a given battery ratio with the training data. If MixMax's aging exceeds that of a single Btype battery, the function returns the aging value as a penalty, whereas it returns the LBT otherwise. Third, we narrow down the search space of the problem according to the constraints derived from S1. Within this constrained space, we find the optimal ratio using an alternating optimization strategy. This iterative process begins with an arbitrary initial value for R_B. We then repeat the following steps 10 times[5] to minimize the objective function: 1) Find the optimal value of R_A/R_C for given R_B and 2) Find the optimal value of R_B for given R_A/R_C. To reduce the time for searching and avoid over-fitting, we limit the number of evaluations for each search as N_E. In each search, Brent's method [34] selects the next battery ratio to minimize the objective function by utilizing the convexity seen in S2.

As a result, the battery ratio optimization with $N_E = 7$ results in finding the ratio that minimizes LBT, which are $R_A = 0.0998$, $R_B = 0.6962$, and $R_C = 0.204$. Although the iterations utilize the training data only, we confirm that LBT of the test data is also efficiently reduced. Also, during the iterations, the return value of the objective function shows convexity, as shown in Fig. 7. The detailed evaluation results will be explained in Section VII.

V. Customized MixMax

In this section, we propose a customized MixMax, aiming to enhance the benefits of MixMax. Our first step is to identify how MixMax benefits individual users, which leads us to group users. Then, we investigate each group's battery usage pattern and finalize the customized MixMax by providing a suitable battery solution for each group.

A. User Categorization by MixMax Benefits

To explore how MixMax benefits individuals, we evaluate its impact on low battery time (LBT) within the framework designed in Section IV. A more detailed explanation of the evaluation method will be presented further in Section VII.

Fig. 8 shows the LBT histogram of 100 users when applying MixMax. Out of 100 users, 35 never experienced the low battery. Among these, 18 users had already never faced low battery states even when using an LCO single-cell battery, while the remaining 17 users got rid of their low battery experiences by adopting MixMax. Note that users who previously had no low battery

[5] 10 iterations were empirically sufficient for convergence.

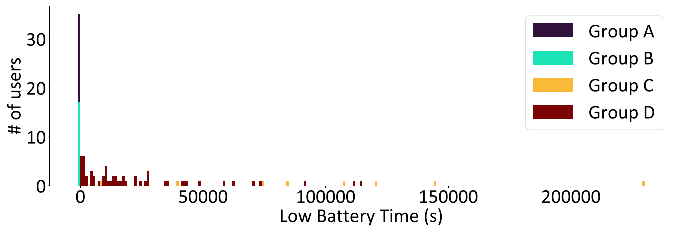
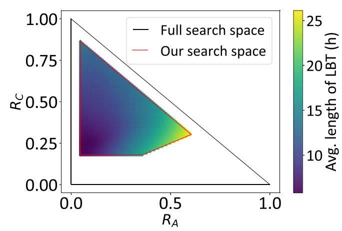
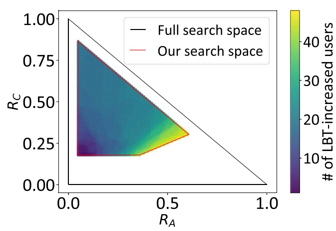


Fig. 8. Histogram of the number of users according to the experienced low battery time when applying MixMax; the leftmost indigo and mint colored bars represent the users who never faced a low battery state.



(a) Average length of LBT results according to battery ratios.



(b) Number of users whose LBT increased according to battery ratios.

Fig. 9. Grid search results for battery ratios among 65 users who still experienced low battery after applying MixMax.

experience with an LCO single-cell battery continued to do so under the MixMax system.

We first categorized users into two groups: Group A, comprising users originally with no low battery encounters, and Group B, consisting of users who no longer faced low battery with original MixMax. In terms of LBT, these two groups do not need additional enhancement. Next, we focus on the 65 users still suffering from low battery. We found that MixMax increased the LBT of 13 out of 65 users. Since our battery ratio optimization approach in Section IV-D minimizes the overall LBT for users in the training dataset, some users may experience an increase in their LBT.

To mitigate this, we performed a grid search for battery ratios and observed the LBT of the 65 users. Although the entire search space is vast, we significantly narrowed it by setting a large search granularity of 0.01 and employing inequalities in Section IV-D as follows:

$$R_A/R_C \leq 2.0, R_A \geq 0.05, R_B \geq 0.09, \text{and } R_C \geq 0.18.$$

Fig. 9 depicts the grid search outcomes, showing (a) the average LBT for users and (b) the number of users whose LBT increased while adjusting battery ratios. Given that the trends in average LBT and the number of users with increased LBT are generally similar across battery ratios, pinpointing an ideal battery ratio could reduce the overall LBT while decreasing LBT for more users. Nevertheless, 8 users could not reduce LBT compared to an LCO single-cell battery for any battery ratio combination. Considering other battery types or a more fine-grained battery ratio search could offer solutions, but these approaches extend beyond the scope of this study. We classified these 8 users into Group C and the remaining 57 users into Group D. The LBT of Groups C and D, as illustrated in Fig. 8, indicate that Group C users experienced much higher LBT.

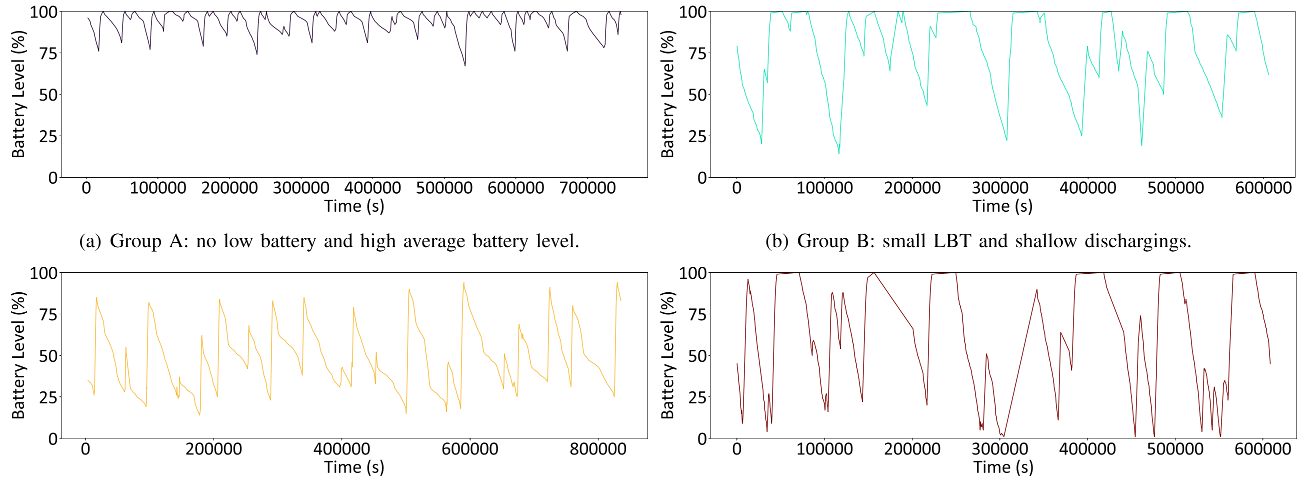
B. *Customizing* MixMax*: Battery Usage Patterns in Focus*

To develop the customized MixMax, a battery solution tailored to different user groups, we analyze the battery usage patterns of each group. Based on this understanding, the customized MixMax provides either original MixMax, a new version of MixMax, or a single-cell battery system to each group.

1) Customized MixMax *for Group A Users:* Users in Group A do not experience low battery, whether using an LCO single-cell or original MixMax. Due to their minimal battery usage, Group A users maintain an average battery level of 77.7%, significantly higher than the 56.9% average of other groups. Fig. 10(a) illustrates the typical battery usage pattern of a Group A user, where the battery level seldom falls below 75%. For this group, prioritizing efforts to reduce low battery time is meaningless. Instead, we leverage the MixMax framework to decelerate their battery aging. We employed the battery ratio optimization methodology from Section IV-D, splitting users into training (70%) and test (30%) datasets to find the ratio that minimizes aging while ensuring zero LBT. The optimized battery ratios were $R_A = 0.3878$, $R_B = 0.3681$, and $R_C = 0.2441$. In minimizing battery aging, the optimization process favored a higher proportion of the long-life A-type battery. For users with low battery usage who do not experience low battery issues, the customized MixMax offers these battery ratios.

2) Customized MixMax *for Group B Users:* Group B users are those who no longer experience any low battery time by adopting original MixMax. Their battery usage pattern has two main characteristics. First, they barely experience low battery. Their average weekly LBT of 1.4 hours was significantly lower than the 10.7 and 12.7 hours of Groups C and D, respectively. Because they faced low battery times only to a small extent, MixMax could completely eliminate their low battery occurrences. Secondly, they undergo shallow dischargings. The Group B users' average depth of discharge (the percentage of battery used per discharge interval) was 34.6%, markedly shallower than the 42.9% and 39.8% of Groups C and D, respectively. Fig. 10(b) depicts the battery usage pattern of a Group B user, who has small LBT and discharges shallowly. The customized MixMax provides original MixMax to users like those in Group B who have shallow low battery experiences and depth of discharge.

3) Customized MixMax *for Group C Users:* The 8 users in Group C showed increased LBT compared to using an LCO single-cell battery for all battery ratio combinations found through the grid search. The cause can be identified in their battery usage patterns, which involve fewer instances of full charging and shorter charging durations. Fig. 10(c) highlights the battery usage characteristics of Group C. Infrequent full chargings limit the utilization of MixMax's C-type battery, and few charging times fail to leverage MixMax's design advantage of securing more energy through charging. Group C's average weekly charging time of 22.1 hours and 2.9 full chargings was notably lower than the 34.9 hours and 6.8 full chargings of users in Groups B and D, making it hard for them to derive the benefits



(a) Group A: no low battery and high average battery level.

(b) Group B: small LBT and shallow dischargings.

(c) Group C: infrequent full chargings and few charging time.

(d) Group D: medium LBT, dischargings, full chargings and charging time.

Fig. 10. Four representative battery usage patterns of each group.

of MixMax. Therefore, customized MixMax provides an LCO single-cell battery for users who rarely charge and seldom fully charge their devices.

4) Customized MixMax *for Group D Users:* Group D overall exhibits a moderate battery usage pattern, as presented in Fig. 10(d). Since Group D includes the largest number of users at 57, it also encompasses the most diverse battery usage patterns among the groups. Therefore, we focused on enhancing MixMax's versatility for these users. We searched for a battery ratio that could reduce LBT for more users. After splitting users into training and test sets, we re-explored the grid search outcomes. Through this process, we identified the battery ratios $R_A = 0.08$, $R_B = 0.72$, and $R_C = 0.19$, which reduce the overall LBT and mitigate low battery experiences for more users. This result demonstrates an optimization towards increasing versatility by incorporating a higher proportion of the moderate-performance B-type battery.

VI. Implementation

We fabricated real A-, B-, and C-type coin-cell batteries and developed accurate enough models of batteries for a proof-of-concept prototype as an emulator. The emulator accurately reflects each cell's physical characteristics (*e.g.*, polarization, internal resistance, or voltage) obtained from the experiment of physical cells, precisely emulating the heterogeneous battery system behavior according to the proposed charge/discharge policies and the battery ratio.

It is a common way of system-level simulation with the battery emulator since the system-level simulation, including electronics and batteries, requires a significant amount of time. The battery emulator is accurate enough to capture the widely known non-linear effects of batteries (*e.g.*, rate-capacity and temperature effects) and is indispensable for evaluating battery system performance over long periods of time (*e.g.*, about 19,855 hours of our battery usage pattern). Attesting to this, battery system studies typically evaluate their system through emulation, for instance, electric vehicles [35], [36], [37], [38], [39], energy storage systems [40], [41], and even mobile

(a) Fabricated coin-cell batteries.

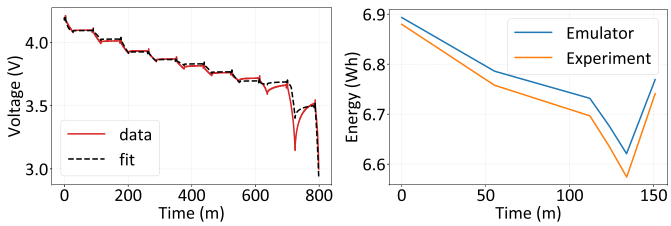
(b) Battery testing equipment.

Fig. 11. Coin-cell batteries for MixMax.

systems [20], [42]. Therefore, we employed the battery emulator for evaluation rather than real-world measurement albeit we implemented MixMax on a demo smartphone, which will be detailed in Section VIII.

We fabricated physical batteries of LTO, LCO, and Li-S in the same form factor for a fair comparison as shown in Fig. 11(a). The cells to be evaluated should have an identical form factor, as the physical characteristics of a battery widely vary according to its form factor. Unfortunately, we could not find our target batteries in the same form factor among commercial off-the-shelf (COTS) ones. The batteries were fabricated in the coin-cell form factor has the size of 2032, the cathode of $1.13\,cm^2$, and the anode of $2.0\,cm^2$. We conducted the *Hybrid Pulse Power Characterization* (HPPC) test to build and evaluate the battery model in a temperature-controlled environment (Fig. 11(b)). We built *Equivalent Circuit Model*, which emulates battery behavior with circuit components and is widely used for long-term battery emulation [43], [44]. The emulator is based on an open-source [45] and incorporates two *Resistor-Capacitor* (RC) networks, which trace the polarization of battery internals to elaborate battery behavior [46]. Note that we also modeled other types of batteries: LTO, LFP, and NCA batteries in the cylindrical form factor for the applicability test.

Finally, the battery emulator shows high accuracy. As shown in Fig. 12(a), the emulator and the physical battery show virtually identical behavior. The emulator makes an average voltage error of up to 1.28%. For each battery cell, the average



(a) Modeled battery emulator of LCO under HPPC test. (b) Emulating MixMax versus real cylindrical batteries experiment.

Fig. 12. Accuracy test of MixMax emulator.

errors are 1.04%, 0.58%, 1.28%, 0.29%, 0.46%, and 0.48% for LTO (coin), LCO, Li-S, LTO (cylindrical), LFP, and NCA, respectively. Fig. 12(b) shows additional experiment result; the energy changes over time of the emulated and physical batteries (composed of LTO-LFP-NCA) with multiple charge-discharge cycles of the identical usage. The emulator shows accurate results only with a 0.3% of energy error on average.

VII. Evaluation

In this section, we evaluate MixMax using the proposed battery emulator in order to demonstrate its effectiveness in reducing low battery time (Section VII-B), the efficacy of the proposed charge/discharge policies of MixMax compared to other approaches (Section VII-C), the applicability of MixMax to other battery systems (Section VII-D), and the benefits of the customized MixMax tailored to diverse user battery usage patterns (Section VII-E).

A. Evaluation Setup

We measured the battery behavior with the emulator while charging/discharging the battery system according to the usage patterns which consist of 19,855-hours-long data collected from 100 users. Note that we divided the data set into 70 training data and 30 test data as described in Section IV-D. The battery ratio parameters (i.e., R_A, R_B, and R_C) were determined based on the training data, and performance measurements were conducted with the test data. While alternating charging and discharging along with the usage patterns, we emulated the battery and measured the performance, including low battery time (LBT), the charging speed, the remaining energy distribution and the battery aging.

It is worth noting how we replay the charge/discharge patterns for different battery systems. While MixMax has the multi-stage charging speed nature, the usage patterns were collected with the single-cell battery. Thereby, it is impossible to replay the charge pattern as is. For a fair comparison, we carefully handle the charge patterns; for each charging interval, we maintain the charging time as in the pattern and recalculate the charging amount according to the charging speed of the battery system. Note that, even in a single-cell battery, the charging speed may vary depending on the charging environment. For instance, the speed may slow down when the device is used during charging, charged at the Constant Voltage (CV) stage, or plugged into a low-power charger. To handle this case, we adjust the charging speed of MixMax for those charging intervals as slowly as the single battery slows down. In contrast, for each discharging interval, we keep the discharging time and rate as they are, since they depend only on the user behavior, not on the battery system.

B. Low Battery Time Reduction

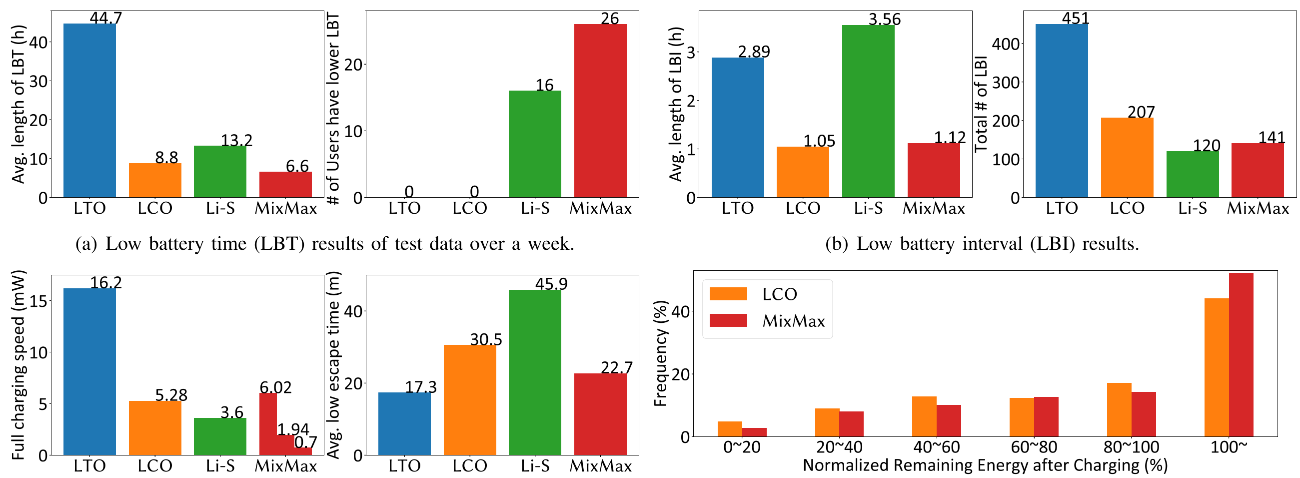
According to the emulation with the usage patterns and the various battery systems (LTO only, LCO only, Li-S only, and MixMax), we have confirmed that MixMax successfully reduces the low battery time (LBT) without changing users' behavior. As the left side of Fig. 13(a) shows, MixMax reduces the overall LBT by 24.6% compared to LCO and exhibits 85.2% less LBT than other single-cell batteries. And the right side of the figure shows that 26 out of 30 users experienced reduced LBT than LCO single-cell.

Two major factors of LBT is the number and length of the low battery interval (LBI). The right side of Fig. 13(b) shows that MixMax reduces the number of LBI by 31.9% than LCO only, which implies that users are less likely to enter the low battery state. And the left side of the figure shows that the average LBI length of MixMax is similar to that of LCO. Despite this similar LBI length, MixMax effectively reduces LBT compared to LCO, because the number of LBI is greatly reduced. In contrast, Li-S shows a worse LBT performance, albeit the lowest number of LBI, because the average LBI length of Li-S is too long due to its slow charging speed.

To further break down the results, we measure how effectively MixMax improves battery performance in terms of the charging speed and energy capacity, and examine how they contribute to reducing LBT. For the charging speed, MixMax has multi-stage charging speed due to the nature of heterogeneous batteries. As Fig. 13(c) shows, the minimum and maximum charging speed of MixMax is 6.02 mWh/h and 0.7 mWh/h, and the average speed for charging a fully discharged battery to its fully charged state is 1.94 mWh/h.

Although MixMax's average full charging speed is slower than LCO, MixMax successfully reduces LBT and escape time (defined by the cumulative length of charging time to escape the low battery state). This implies that our discharge policy well exploits the multi-stage charging speed so that it maximizes the benefits of maximum charging speed, while minimizing the drawback of minimum charging speed. Indeed, we have confirmed MixMax's average remaining energy after each charging/discharging is 16.7% higher than that of LCO. Furthermore, although MixMax's maximum charging speed is only 14.1% higher, its average time to escape LBI is 25.7% shorter, compared to those of LCO. This is because the amount of energy required to escape the low battery state itself has decreased due to the benefit of improving both charging speed and capacity.

As for the capacity, MixMax supports 15.2% more energy than the corresponding same-volume LCO battery without compromising any other usability, which is a staggering achievement that would take about seven years ([47], [48], 2011-2018, 14.7%) to advance in battery material alone. The capacity and charging speed increase leads to the remaining energy increase. Fig. 13(d)



(a) Low battery time (LBT) results of test data over a week.

(b) Low battery interval (LBI) results.

(c) Advantage of MixMax in escaping the low battery state.

(d) Battery level histogram normalized by LCO capacity.

Fig. 13. Evaluation of MixMax compared to single-cell battery systems.

shows the distribution of the remaining energy after charging intervals. It shows that the remaining energy of MixMax is distributed in the higher range compared to LCO. Note that the remaining energy distribution of MixMax is not simply right-shifted from that of LCO; instead, MixMax effectively reduces the lower range of the remaining energy distribution, in particular, less than 20%. This result confirms that MixMax is effective in rapidly escaping LBI and keeping more energy, compared to the single-cell LCO battery.

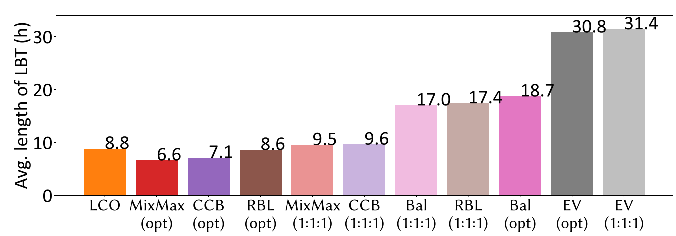


Fig. 14. Low battery time reduction of MixMax according to discharge policies and battery ratios.

C. Comparison Study

We compare components of MixMax with other design candidates to investigate how each component contribute to reducing LBT. In comparison, we target the intricately designed discharge policy and battery ratio optimization.

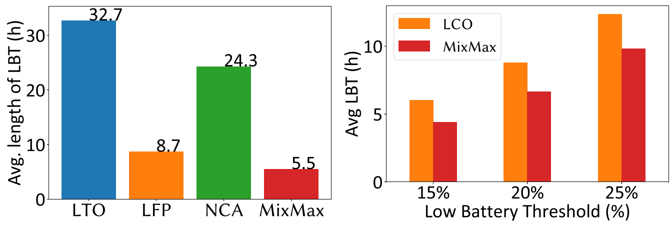
We first compare the discharge policy with the following other policies from recent work. Software-defined battery (SDB) [20] proposes two discharge policies aiming for mobile systems; one to balance the aging of each battery type (`CCB`) and another to minimize the energy loss from internal resistance (`RBL`). Multi-cell battery systems generally adopt a discharge policy for cell balancing (`Bal`) [23], which equalizes the remaining energy of multiple batteries, and for EVs, a policy proposed (`EV`) prioritizing to discharge a battery with the smallest power performance (i.e., charge speed and maximum power output) to prepare peak power load. Then, we compare the case where the battery ratio is optimized for each discharge policy as MixMax does in Section IV-D (`opt`) and the case used in a naive 1:1:1 ratio (`1:1:1`). Fig. 14 shows the overall results. From the figure, we make the following two observations.

First, as shown in Fig. 14, when using our MaxiMin discharge policy, users undergo the shortest LBT. A user experiences an average of 6.6 hours of LBT over a week when employing MixMax (`opt`), which fully exploits MixMax 's designs, while all other approaches show worse results. Since our discharge policy is designed to maximize the charging speed of the upcoming charging intervals, the ratio of charge intervals ending up with full capacity should be larger than other policies. Applying our policy comes up with the highest ratio, i.e., 29.5%, of the charge intervals in which the battery is fully charged for (`opt`) case. For all other policies, this ratio shows the opposite trend to the average LBT, as expected. `CCB` shows a ratio of 23.9%, which is higher than the remaining ones, and all the other policies show a lower ratio of 19.7%. It is trivial that the more frequently the battery is fully charged, the less likely users suffer from LBT.

Second, the LBT reduction of ratio optimization depends on the discharge policy. For example, in Fig. 14, our battery ratio optimization halves the LBT a user experiences over a week when using the `RBL` discharge policy, whereas it rather increases by 9.8% in the case of `Bal`. The **S3** step of our battery ratio optimization searches the optimal battery ratio minimizing LBT subject to less aging than LCO single-cell battery. The discharge policies `Bal` and `EV` cause the most battery aging because they utilize the C-type battery more than other discharge policies. Thus, the optimization processes of `Bal` and `EV` have no room to minimize LBT as they allocate much A-type battery ratio to lessen aging than LCO single-cell. This indicates that ratio optimization cannot effectively minimize LBT for bad discharge policy and endorses our solution approach of ratio optimization after designing the discharge policy first in Section III-B.

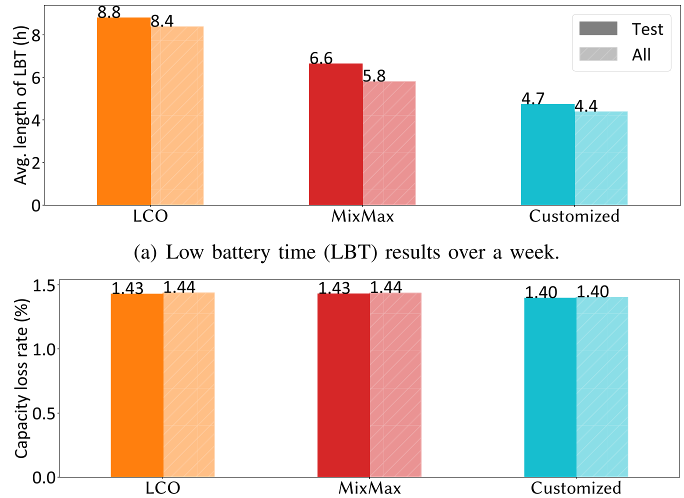
D. MixMax *in Other Environments*

Other battery types: Thanks to the general design of MixMax, it is possible to make up MixMax with other battery types, for



(a) Different battery types. (b) Different low battery threshold.

Fig. 15. Average low battery time of users over a week with different battery types and low battery threshold.



(b) Battery aging results (capacity loss rate) over a week.

Fig. 16. Evaluating customized MixMax against LCO and original MixMax.

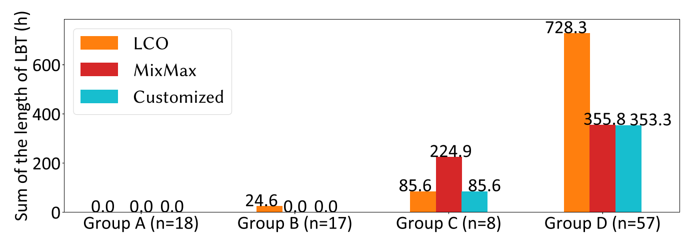


Fig. 17. Group-wise total LBT evaluation of LCO, original MixMax, and customized MixMax.

example, LTO, LFP and NCA, although they are mainly used in other than the mobile environment. To verify that our approach is applicable to other environments, we composed MixMax* with 18650 cylindrical cells of the other battery types, in particular, LFP, LTO, and NCA as A-, B-, C-types, respectively. Afterward, we again emulated MixMax* to measure LBT. As shown in Fig. 15(a), MixMax* still shows better performance with regard to LBT compared to other single-cell batteries.

Various low battery thresholds: Changing the low battery threshold also changes LBT that users experience. To show how the threshold affects LBT, we vary the low battery threshold from 15% to 25% of the total amount of energy, and measure the average LBT. Except for the low battery threshold, all other evaluation environments are the same as in Section VII-B. Fig. 15(b) shows that even with different low battery thresholds, employing MixMax always achieves lower LBT than LCO battery. For instance, when the threshold is set to that of Android (i.e., 15%), users experience an average of 6.0 and 4.4 hours over seven days with LCO battery and MixMax, respectively.

E. *Evaluating Customized* MixMax

To evaluate the effectiveness of customized MixMax, we compared it with both LCO and original MixMax. This evaluation focused on assessing low battery time (LBT) and battery aging (capacity degradation). We extended our evaluation beyond the test set to evaluate the entire dataset (i.e., training + test) in order to compare performance across different groups.

As depicted in Fig. 16(a), customized MixMax significantly decreases LBT in comparison to LCO and original MixMax. Specifically, customized MixMax reduced LBT by 46.2% and 28.6% compared to LCO and original MixMax, respectively, in test data evaluations, and 47.7% and 24.4% on all datasets. To understand these LBT reductions, we examined the group-specific LBT results. Fig. 17 shows the total LBT by group, indicating that groups with more users exhibited higher cumulative LBT. For Groups B and C, where customized MixMax adopted the original MixMax and LCO single-cell, customized MixMax achieved low LBT by incorporating the advantageous battery system. Notably, in Group C, whereas original MixMax significantly increased LBT over LCO, customized MixMax effectively avoided this increase by utilizing LCO, significantly reducing overall LBT.

The newly optimized battery ratios by customized MixMax also achieved good results. In Group A, it ensured that no users experienced low battery levels, and in Group D, it exceeded the performance of the original MixMax by further reducing LBT. Another noteworthy point is that the original MixMax caused 13 out of 100 users to experience an average LBT increase of 11.6 hours compared to LCO, whereas customized MixMax led to only 2 users with an average 4.9 hours increase over LCO. This indicates that customized MixMax enhances MixMax's benefits in terms of LBT but also drastically reduces both the number of disadvantaged users and the extent of their disadvantage.

Customized MixMax also effectively reduced battery aging. Fig. 16(b) shows the percentage reduction in battery capacity (capacity loss rate) due to aging over an average of 1 week, with lower values being better. For all dataset, customized MixMax reduced aging by 2.4% and 2.3% compared to LCO and the original MixMax. This contrasts with the original MixMax, which minimizes LBT while maintaining aging. By dividing into groups, customized MixMax's aging reduction can be seen more dramatically. Fig. 18 shows the total battery aging for each group. In the case of Group D, customized MixMax slightly increased aging compared to LCO, which seems to be a small difference caused by the mismatch between the training and test sets. With the exception of Group D, customized MixMax always showed less aging than LCO and the original MixMax. In

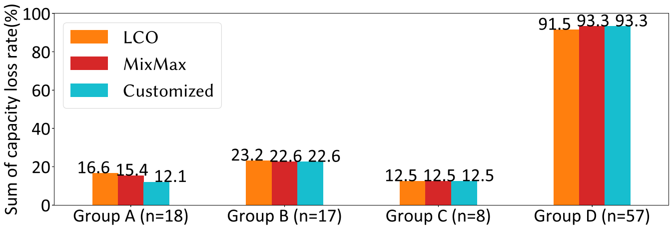


Fig. 18. Group-wise total aging evaluation of LCO, original MixMax, and customized MixMax.

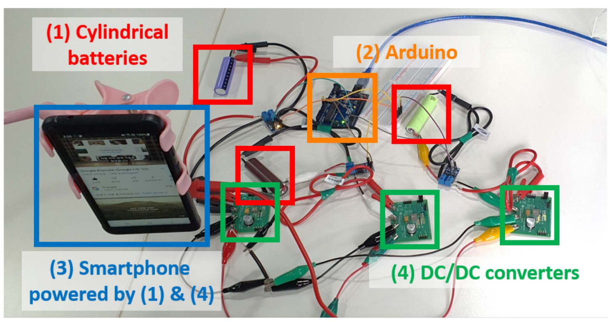


Fig. 19. Implementation of MixMax on a demo smartphone.

particular, for Group A, which was optimized to minimize aging, aging progression was reduced by 27.5% and 21.6% compared to LCO and original MixMax, respectively. This demonstrates the possibility that the MixMax methodology can be actively used to improve problems in domains other than low battery.

VIII. Field Test: A Demo Smartphone

In this section, we demonstrate the practicality of MixMax as a demo smartphone field test. A heterogeneous battery system, unlike a single-cell battery system, requires switching and voltage conversion of batteries. Thus, MixMax can be commercialized only when the switching and converting require affordable physical costs and ensure system stability. Our field test on the smartphone addresses these concerns.

Setup details: Fig. 19 depicts the prototyped demo smartphone. We applied MixMax on a smartphone named SM-G525 N with 18650-sized cylindrical LTO, LFP, and NCA batteries (corresponding to A-, B-, and C- type). Note that we used cylindrical batteries instead of fabricated coin-cell batteries for sufficient power output. As the COTS smartphone regulates its input voltage (although it already converts voltage internally), we added DC/DC converters to meet the input voltage requirement. The circuit topology was designed based on Section IV-A; each battery powers the smartphone through each DC/DC converter, and the microcontroller controls the usage of batteries (i.e., discharge policy) by switching batteries. TPS61022EVM-034, Arduino UNO, and MOSFET switches were used for DC/DC converter, microcontroller, and turning on/off batteries, respectively, and the battery switching granularity is in the order of milliseconds.

Operation stability: The demo smartphone is stably powered by heterogeneous batteries according to the design of MixMax. We test the operation of the demo smartphone by booting, running the YouTube app for an hour, and turning off the device. In the test, the demo smartphone operates without any failure, even during switching, and fully follows MixMax's MaxiMin discharge policy.

Energy loss: One may worry that the additional circuits of heterogeneous battery systems incur significant energy dissipation, degrading performance. We measure and find that circuits for three batteries require 1.57% more power than those for one battery due to energy loss. Considering we handmade the demo and only used one type of DC/DC converter, the loss will be much lower in practice. SDB [20] confirms that the energy loss from the additional circuits of heterogeneous batteries is no more than 1%, although their circuits support complicated operations such as energy migration between batteries. As MixMax does not require a circuit as complex as SDB, we can assume that MixMax's energy loss is less than 1% in practice. Even if we harshly assume there is always a 1% loss during usage, the LBT of MixMax is still 9.7% less than the single-cell LCO battery.

Additional costs for required parts: Looking at Fig. 19, MixMax seems to require many massive and costly parts, which is not true in practice. Firstly, the demo smartphone looks huge just because we used bulky ready-made boards of the DC/DC converters for ease of implementation. The sole volume of the DC/DC converters themselves is smaller than 4 mm^3 [49], and their weight and price are also negligible, around 43mg and 0.6$, respectively. The volume, weight, and size of other parts are even smaller.

In addition, the costs associated with the required parts can be minimized during the production stage. One way to achieve this is by placing these parts in empty spaces inside smartphones, taking advantage of their tiny sizes. Another way is to replace the required DC-DC converters with existing converters that are already included in modern smartphones. Note that modern smartphones have some empty spaces [30], [31] and include several tens of converters that are versatile in dealing with multiple sources of charge in various conditions [31], [32]. The selection process of heterogeneous batteries can also be integrated into the power path without incurring much extra cost. For instance, a multiple-input DC-DC converter selects the proper battery without extra input power gates. Such integration helps cut cost and size through better MixMax hardware design.

IX. Related Work

Improving battery performance for mobile devices without battery change: A large body of research has studied techniques to improve the performance of batteries for mobile systems. One of the most intuitive ways to improve battery performance is to reduce energy consumption. These studies propose the energy consumption reduction of video streaming [8], file downloading [14], web interactions [50], [51], and image sensing [52], in addition to hardware usage like CPU [53], GPU [54], [55], GPS [56], [57], network [15], and display [6], [58]. Also, these techniques span a broad spectrum of aspects, including but not limited to, reducing energy consumption of measurement and analysis techniques [5], [7], [9], [10], [11], [12], charging techniques [42], [59], [60], [61], and energy management at both the application [5], [7], [13], [16], [17] and system levels [18], [62]. Recently, Tang et al. [2], [63] alleviated the low battery anxiety of mobile users' using a technique of low power video streaming. However, all of the above studies considered single-cell battery, and most of them compromised users' behavior. Therefore, the studies handle totally different layers from our research, which enables to apply them in parallel with MixMax.

Heterogeneous batteries in other applications: Many different batteries have their own advantages and disadvantages in terms of various performance metrics. A few studies have developed heterogeneous batteries to exploit the advantages of different batteries while hiding their disadvantages. They have mainly focused on the development of hardware and software components [64], circuit topology [65], and control and management strategies [66], [67]. However, none of the above studies considered the usage characteristics of mobile devices and aimed at minimizing the low battery time, which are the focus of this paper.

Battery change for mobile devices: Software-Defined Batteries [20] developed an operating system and circuit for heterogeneous batteries and proposed two discharge policies for general purposes. A multi-cell system [19] utilizes homogeneous multiple batteries of different shapes and sizes to form a large battery, while cooling-sensing battery management [21] simulates cooling and power demand to optimize the use of heterogeneous big.LITTLE batteries. However, they do not consider the battery ratio, and the all their performance improvement only comes from discharge intervals. As we demonstrated from design and evaluation of MixMax, the charge and discharge intervals and the battery ratio should all be considered for low battery anxiety; as a result, those studies are not effective in reducing the low battery time.

Studies on low battery experience: A wide range of research across various fields has been conducted to define the low battery experience. These studies investigate what users experience when their battery is low, and how users perceive the low battery experience. In the field of engineering, Tang et al. and Zhang et al. have modeled the degree of low battery anxiety felt by mobile users depending on their battery level [63], [68]. On the other hand, in the fields of medicine and psychology, Bragazzi and Del Puente have proposed the inclusion of nomophobia in the fifth version of the Diagnostic and Statistical Manual of Mental Disorders (DSM-V), which is a standard manual for assessing psychiatric diseases [69]. Furthermore, Yildirim and Correia developed a questionnaire to measure nomophobia [70].

X. Discussion

Integration with low power management software: Mobile OSes already manage the low battery situation by power saving modes [71], [72], [73], and there are many power-saving software techniques [7], [14], [15], [16] which limit background services, CPU, or screen. Since such approaches are orthogonal to MixMax, we can further improve the low battery experience by adopting those software *and* MixMax at the same time. And if the device driver or scheduler can be aware of the multi-stage charging speed of MixMax, they may offer more advanced charge/discharge policies taking the user and system contexts into account. We leave it as future work.

Users' battery usage patterns: Our evaluation method of replaying users' battery usage patterns is reasonable. MixMax will not change the user behavior much as it does not change the battery much, e.g., a 15% increase in capacity. Our battery usage pattern data find there are very small correlations [74] between the maximum battery capacity and key battery usage patterns, such as average charging time, charging trials, and discharging amount. This is because the key patterns mainly depend on the usage situation (e.g., charging during sleeping) rather than the battery itself.

However, if battery capacity or charging speed changes significantly beyond our dataset, it may alter user behavior. Understanding and leveraging such changes requires further study, which we leave as future work.

CCCV (Constant Current Constant Voltage) charging: One might think that MixMax did not fully consider the slowing down of the charging speed caused by the constant voltage (CV) phase of CCCV charging. However, the effect of MixMax on the CV charging is robust. The proposed rate-based discharging strategy is not affected by CV charging because it calculates the full charging time while taking into account the effects of CV charging. Although the t_1 and t_2 equations derived in Section IV-D do not explicitly consider the CV charging, they can calculate t_1 and t_2 robustly. When R_A/R_C is 1, the calculated t_1 and t_2 are 1,417 seconds and 8,221 seconds, respectively, and the actual t_1 and t_2 are 1,358 seconds and 8,218 seconds.

Impact of a charging policy: Slow battery charging speed can help decelerate battery aging. There have been many studies [42], [59], [60], [61] decelerating battery aging by slowing down the charging speed. Thanks to these studies, modern smartphones now employ charging slowdown and charging delay during sleeping hours to decelerate battery aging, like Apple's optimized charging [75]. Future work can retrofit MixMax's charging policy with a better one that leverages the advantages of the slow charging speed.

Other constraints—price and weight: While MixMax considers volume, capacity, and aging as optimization constraints, other factors such as weight and price would be important for someone. Although our optimization framework does not explicitly consider these factors, we found that the weight and price of MixMax are -0.4% and 9.48% higher than those of LCO, respectively. To determine this, we calculated the price of cathode, anode, electrolyte, separator, and coin-cell cases per one coin-cell battery and measured the average weight of each battery. For reference, the price and weight of the fabricated single LTO, LCO, and Li-S coin-cells are approximately $5.380, $3.946, and $ 5.079, and 3.7365g, 3.7526g, and 3.7365g, respectively. Note that the increase in the price would be affordable since the price of a battery possesses only a small portion (i.e., 1.4% [31]) of the cost of a smartphone and such price differences can be similar for other battery form factors because the material costs account for more than 76% of the total battery production costs [76].

Thermal impact: The battery is deeply related to the thermal behavior of a smartphone. The battery can generate heat from internal resistance and influence the phone's thermal management. Moreover, a smartphone battery is protected from overheating, which can also affect its thermal behavior. The battery's internal resistance, as well as AP, Wi-Fi, LTE, and GPS [77], contributes to the heat generation of a smartphone. Yet, the amount of heat from the battery is so tiny that it is generally not considered when assessing a smartphone's thermal characteristics [77]. This holds true even for newer battery chemistries being adopted by MixMax, like LTO and Li-S. Therefore, changes in battery type due to MixMax have a negligible impact on the overall thermal behavior of a smartphone. Because the amount of heat generated by the battery is low regardless of the battery type, it is possible to avoid accidents without altering the temperature threshold (e.g., $68^{\circ}C$ [77]) managed by throttling.

Conversion loss: The addition of DC/DC converters for MixMax could raise concerns about increased energy loss during conversion, as each battery type has a unique voltage.

However, the energy loss of a DC/DC converter is determined by its input voltage (from the battery) and the output voltage and current load (from components like the CPU, display, and Wi-Fi chip). To operate efficiently under varying conditions, mobile systems already integrate multiple DC/DC converters and select the one with the least energy loss based on the system's load and the battery's state. Similarly, a system equipped with MixMax can select the appropriate DC/DC converter for each battery type and operating condition, ensuring that no significant additional energy loss occurs during the conversion process.

Size of dataset: The use of a 100-user dataset may raise concerns about overfitting. Although a 13% LBT difference was observed between training and test sets, it does not significantly affect our key findings. The limited dataset also caused some overlap between training and evaluation data for customized MixMax. We believe that expanding the dataset is an important future direction to further mitigate overfitting and better validate the customized MixMax.

XI. Conclusion

We present MixMax, a heterogeneous mobile battery system that mitigates the low battery experience. MixMax develops the charge & discharge policies for the three different battery types and determines the battery composition ratio, achieving LBT minimization, which is demonstrated by the precise battery emulator based on fabrication of coin-cell batteries and field test. We expect MixMax to evolve in various directions such as aging factor, predicting usage patterns, expanding data sets and integrating with OS, which we leave as future work.

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